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Physics Letters A 308 (2003) 391–396
PHYSICS LETTERS A
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Noise-induced order phenomenon in semiquantum system

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Received 18 October 2002; received in revised form 3 January 2003; accepted 12 January 2003

Communicated by A.R. Bishop

Abstract

Using a semiquantum system, we have studied the influence of different variance white-noise. NIO have been found taking place in the system. Suitable noise can turn the system from chaotic evolution to regular evolution, and the final trajectory may appear more stable with weak noise applying than pure regular ones. A simple explanation has been made to explain this phenomenon.

PACS: 05.45.-a; 03.65.Sq; 03.65.-w

Keywords: Noise-induced order; Semiquantum chaos

1. Introduction

Noise-induced order (NIO) phenomenon has become a hot topic in recent years. As a cursory but straightforward consequence of the second law of thermodynamics, the noise applied on a system always tends to disorder the system. Noise has been known to play a detrimental role in many experimental situations, which made people to develop better techniques and methods to minimize, if not totally remove, the effect of noise and enhance signal-to-noise ratio and hence system performance. But people have found [1–3] in some cases the noise can play a counterintuitive role, i.e., some system will turn to a more ordered state with right noise, which was first reported, e.g., in the computer simulation done by Matsumoto and Tsuda [4]. And it is observed that noise can trigger, select and sustain patterns in optical systems, fluid dynamical systems, etc. [5–9]. But these studies only pay attention to the behavior of classical NIO phenomena and a complete and convincing theory has not been set up to explain them. A natural next step is to see how the noise influences quantum systems, and if this kind of NIO phenomenon can be observed in quantum situations. Of course, the absent of any direct counterpart to classical trajectories in phase space for a quantum system poses a special problem in the non-linear dynamical system from the point of view of the quantum-classical correspondences [10].

In this Letter, for simplicity, we will consider a *semiquantum* system in which a classical oscillator interacts with a purely quantum-mechanical oscillator.

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0375-9601/03/$ – see front matter
doi:10.1016/S0375-9601(03)00079-3

This system was first considered by Cooper et al. [11,12], which can be described by a classical effective Hamiltonian, obtained from the expectation value of its quantum Hamiltonian. We will add a weak white-noise in this so-called *semiquantum chaos* system, and observe whether the NIO will happen in it.

This Letter is organized as follows: in Section 2, we will introduce the basic model about the system. In Section 3, NIO in the *semiquantum chaos* system is discussed. Finally, we will end our Letter with some conclusions and discussions in Section 4.

2. Semiclassical description of the coupled harmonic oscillators

In this Letter, we mainly deal with a couple system, composed of a classical oscillator and a purely quantum oscillator, which is described by a Hamiltonian [11]

$$H = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\dot{A}^2 + \frac{1}{2}\left(m^2 + e^2A^2\right)x^2, \tag{1}$$

where the coordinates x and A describe, respectively, the motion of the quantum oscillator and the classical one in the system. Faccioli et al. in their comment [12] gave a more proper treatment for this system. With their method, we can describe the system with two coupled equations [13]

$$\ddot{A} = -\frac{\partial}{\partial A}\langle \widehat{H}_x\rangle_n = -(2n+1)\hbar e^2 AG, \tag{2a}$$

$$\frac{1}{2}\frac{\ddot{G}}{G} - \frac{1}{4}\left(\frac{\dot{G}}{G}\right)^2 - \frac{1}{4G^2} + m^2 + e^2A^2 = 0. \tag{2b}$$

Here $G = \langle x^2\rangle_0$, n is a integer, and for an operator $\widehat{O}$, its average value in nth excited state is defined as

$$\langle \widehat{O}\rangle_n = \langle n|\widehat{O}|n\rangle. \tag{3}$$

Here $|n\rangle$ is the nth excited state wave function of the quantum oscillator,

$$|n\rangle = \left(\frac{1}{2G\hbar}\right)^{1/4} H_n\left(\frac{x}{\sqrt{2G\hbar}}\right) \times \exp\left[-\frac{1}{4G\hbar}\left(1 - i\dot{G}\right)x^2\right], \tag{4a}$$

where $H_n(x)$ is the nth Hermit function, defined as:

$$H_n(x) = (-1)^n\left(2^n n!\sqrt{\pi}\right)^{-1/2} \times \exp\left(x^2\right)\frac{d^n}{dx^n}\exp\left(-x^2\right). \tag{4b}$$

And it is easy to get the quantum part energy and the total energy:

$$E_{Qn} = (2n+1)\hbar\left[\frac{\dot{G}^2+1}{8G} + \frac{1}{2}\omega^2 G\right], \tag{5a}$$

$$E_n = \frac{1}{2}\dot{A}^2 + E_{Qn}. \tag{5b}$$

3. NIO in the semiquantum chaos system

We add a Gaussian white noise $\Gamma(t)$ into the classical part of the system,

$$\ddot{A} = -(2n+1)\hbar e^2 AG + \Gamma(t), \tag{6a}$$

$$\frac{1}{2}\frac{\ddot{G}}{G} - \frac{1}{4}\left(\frac{\dot{G}}{G}\right)^2 - \frac{1}{4G^2} + 1^2 + e^2A^2 = 0, \tag{6b}$$

where

$$\left\langle \Gamma(t)\right\rangle = 0, \qquad \left\langle \Gamma(t)\Gamma\left(t'\right)\right\rangle = D^2\delta(t-t'). \tag{7}$$

For simplicity, we will use the units of $\hbar = 1$ and $m = 1$.

Here, we change D, and calculate the Lyapunov exponent (LE) λ. To show the time evolution of λ, a short time Lyapunov exponent $\lambda(t, \Delta t)$ has been defined as follows.

Suppose a dynamic system is described by a trajectory $r(t)$ in m-dimensional phase space, i.e., $r \in R^m$. Let $r(t) + \delta r(t)$ is an arbitrary trajectory nearby, then the short time Lyapunov exponent (STLE) $\lambda(t, T)$ is

$$\lambda(t, T) = \frac{1}{T}\ln\left(\lim_{|\delta r(t)|\to 0}\frac{|\delta r(t+T)|}{|\delta r(t)|}\right), \tag{8}$$

where T is some period of time, which is selected to be about 180 periods of quantum part oscillator in our system (and if it is selected differently, we can still get similar results). Obviously, we can get

$$\lambda = \lim_{T\to\infty}\lambda(t, T). \tag{9}$$

Given the same total energy $E = 0.8$, we use the initial conditions of Eq. (6) as the following:

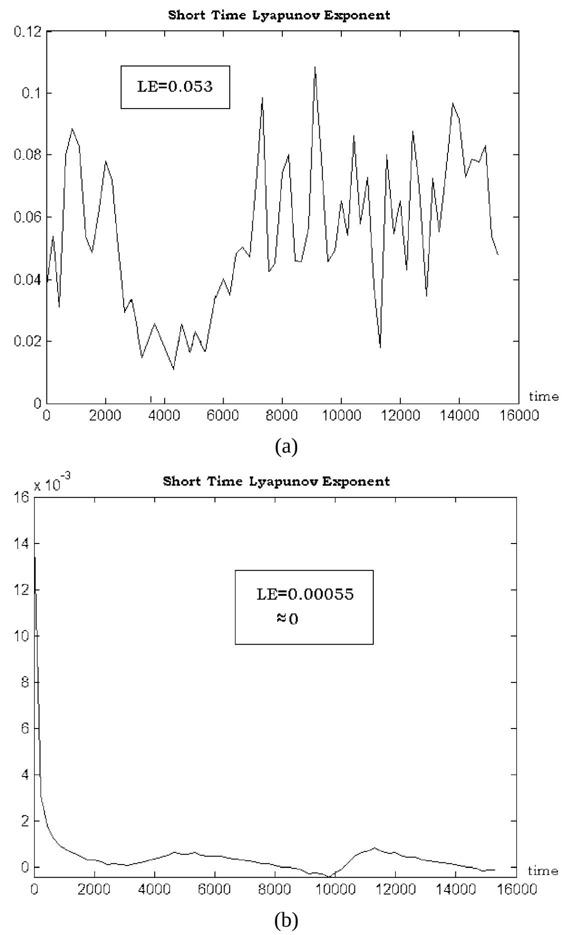

Fig. 1. Evolution of STLE when $D = 0$. (a) Initial chaotic state. (b) Initial regular state.

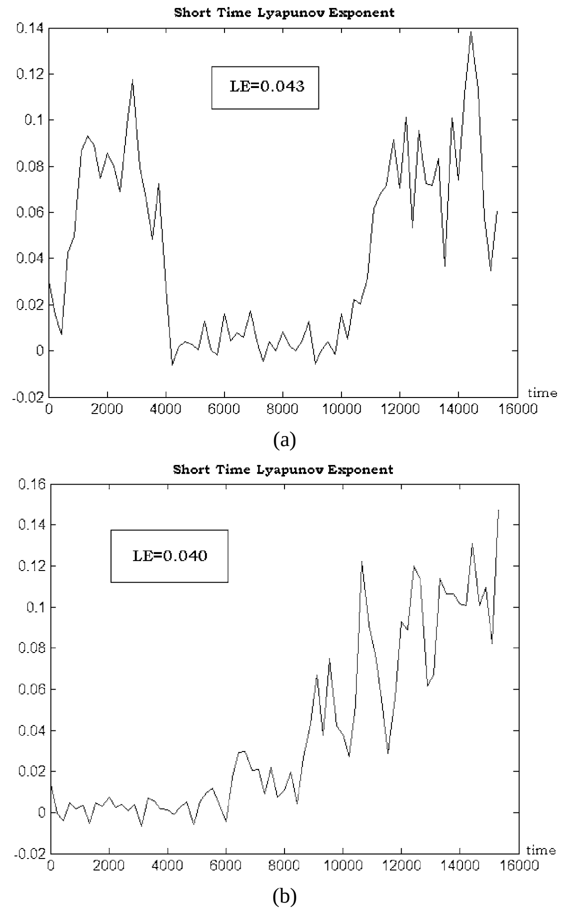

Fig. 2. Evolution of STLE when $D = 0.001035$. (a) Initial chaotic state. (b) Initial regular state.

(i) Chaotic state: $A = 0.0$; $G = 0.35$; $\dot{A} = 0.0$; $\dot{G} = 0.731925$; $e = 1.0$;
(ii) Regular state: $A = 0.0$; $G = 0.5$; $\dot{A} = 0.0$; $\dot{G} = 0.774597$; $e = 1.0$.

These values are chosen as the same as those in Ref. [13].

The evolution of STLE with change of D is shown below, and the insets shows the LE of total evolution.

From Fig. 1, we can clearly see that with the two initial conditions the system evolves in chaotic and regular state, respectively. In the regular case, the STLE turns to be near zero in about 2000 s, but the STLE keeps a non-zero value in the chaotic case.

Fig. 2 is got by setting $D = 0.001035$. In Fig. 2(a) we can find that the noise makes the STLE to keep a small value (near zero) in a long period of about 6000 s (4000 s to 10000 s). In other words, the intermittent NIO can happen in this system. While the same noise applied on regular state (Fig. 2(b)) makes the system to be more chaotic.

Amazingly, when $D = 0.00104$, the STLE falls down to zero in a short time, and keeps a very small value in the following evolution (Fig. 3(a)). Again, the same noise makes initial regular state evolve into chaotic in part time (Fig. 3(b)). And the LE of this evolution is much smaller than other evolutions.

Fig. 4 is got by setting $D = 0.001045$. Roughly, Fig. 4 looks very similar to Fig. 2, and intermittent NIO happens in the chaotic system.

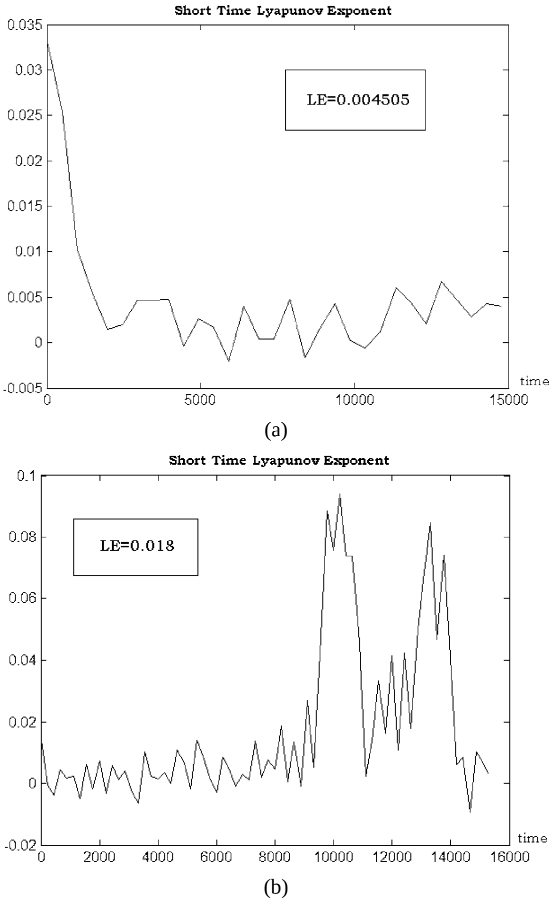

Fig. 3. Evolution of STLE when $D = 0.00104$. (a) Initial chaotic state. (b) Initial regular state.

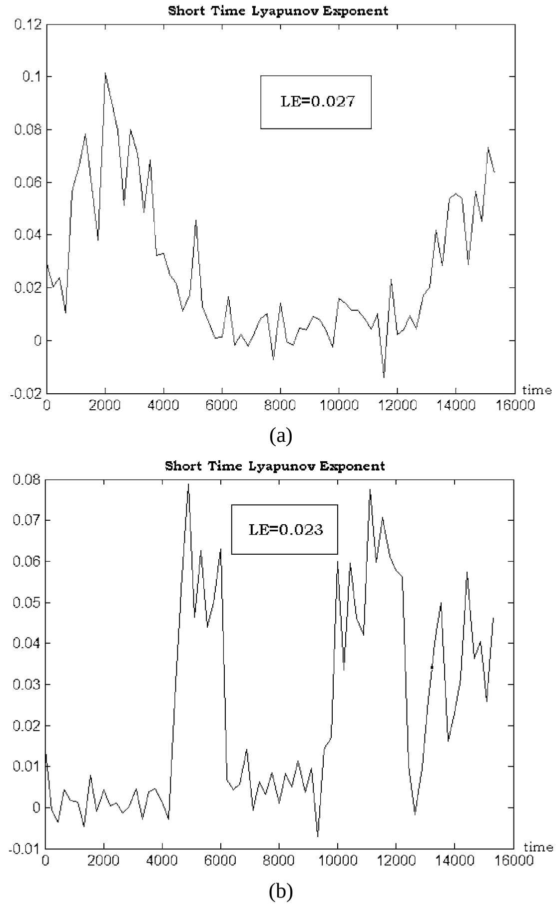

Fig. 4. Evolution of STLE when $D = 0.001045$. (a) Initial chaotic state. (b) Initial regular state.

From above four figures, we find in many cases, the intermittent NIO phenomenon can be found in the initial chaotic situation with a weak noise. And if the variance of the noise is chosen appropriately, the chaotic evolution can be restrained totally. On the other hand, the noise applied on the regular state will disturb its evolution, making the normal regular state unstable in the most cases.

But there is a question: is there any common attribute among these NIO state? Here we give a simple discussion.

We make Fourier transformation of the probability density $|\phi|^2$, where ϕ is the wave function of the quantum part in the ground state, and we only study the segment where NIO takes place (when $D = 0$, we take regular one instead). Obtained results are shown in Fig. 5, we can find Fig. 5(a) and (b) are very similar, while Fig. 5(c) and (d) are very similar, too, but they are all very different from Fig. 5(e), which shows the pure chaotic evolution without the noise. With criterion presented in Ref. [13], we can easily get a conclusion that Fig. 5(a)–(d) is in regular evolution. Fig. 5(a) and (b) have the similar '*fundamental frequencies*', and those peaks in Fig. 5(a) is much sharper than those in Fig. 5(e), because of which reason, Fig. 5(a) can be regarded as a very good regular evolution, in other words, the chaotic evolution has been suppressed. The similar phenomenon in the classical system has been reported by Brainman et al. [14]. It seems that the noise can 'choose' some special trajectories. In fact,

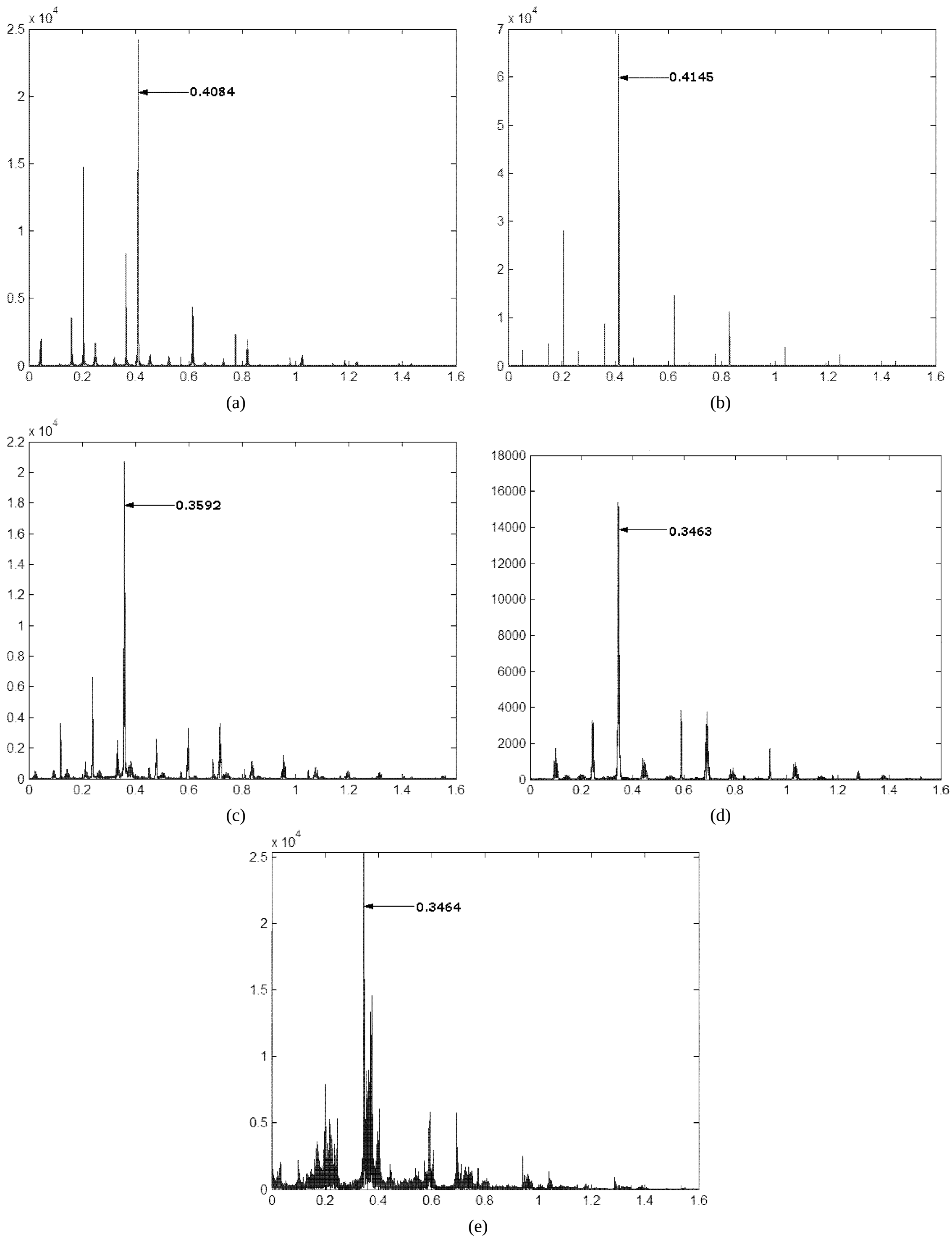

Fig. 5. Fourier spectra (FS) of the probability density $|\phi|^2$ when NIO happens. (a) $D = 0.00104$, initial chaotic. (b) $D = 0$, initial regular. (c) $D = 0.001035$, initial chaotic. (d) $D = 0.001045$, initial chaotic. (e) $D = 0$, initial chaotic.

KAM theorem [15] has implied that, chaos can only take place in some very small regions of phase space when weak disturbance applied on a nearly integralable system. That is to say, when weak noise (a kind of weak disturbance) exists, there must be some regular evolutions which are more stable than chaotic ones.

4. Conclusions

In this Letter, we applied a weak white-noise on a semiquantum system, changing variance of the noise. We compared the evolution of STLE with different initial conditions, and find that suitable noise can turn the system from chaotic evolution into a more stable regular evolution. Increasing or decreasing the variance, intermittent NIO will happen in this system.

References

[1] K. Wiesenfeld, F. Moss, Nature (London) 373 (1995) 33;
F. Moss, K. Wiesenfeld, Sci. Am. 273 (8) (1995) 50.

[2] M.I. Dykman, D.G. Luchinsky, R. Mannella, P.V.E. McClintock, N.D. Stein, N.G. Stocks, Nuovo Cimento D 17 (1995) 661.

[3] H. Yamazaki, T. Yamada, S. Kai, Phys. Rev. Lett. 81 (1998) 4112.

[4] K. Matsumoto, I. Tsuda, J. Stat. Phys. 31 (1983) 87.

[5] F. Moss, P.V.E. McClintock (Eds.), Noise in Dynamical Systems, Vols. 1–3, Cambridge Univ. Press, Cambridge, UK, 1989.

[6] F. Moss, L.A. Lugiato, W. Schleich (Eds.), Noise and Chaos in Nonlinear Dynamical Systems, Cambridge Univ. Press, Cambridge, UK, 1990.

[7] D.A. Kurtze, Phys. Rev. Lett. 77 (1996) 63.

[8] R.J. Deissler, Phys. Lett. A 120 (1987) 334.

[9] H.R. Brand, R.J. Deissler, Phys. Lett. A 130 (1988) 293.

[10] M. Gutzwiller, Chaos in Classical and Quantum Mechanics, Springer-Verlag, Berlin, 1990;
I.C. Percival, Adv. Chem. Phys. 36 (1977) 1.

[11] F. Cooper, J.F. Dawson, D. Meredith, H. Shepard, Phys. Rev. Lett. 72 (1994) 1337.

[12] L. Faccioli, F. Finelli, G.P. Vacca, G. Venturi, Phys. Rev. Lett. 81 (1998) 240.

[13] Y.W. Hui, J. Dong, D.Y. Xing, Phys. Rev. E 62 (2000) 6318.

[14] Y. Braiman, I. Goldhirsch, Phys. Rev. Lett. 66 (1991) 2545.

[15] A.J. Lichtenberg, M.A. Liebermann, Regular and Stochastic Motion, Springer-Verlag, New York, 1983.